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Monetary Stability and Liquidity Crises: The Role of the Lender of Last Resort

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M E X I C O

Monetary Stability and Liquidity Crises: The Role of the Lender of Last Resort[¤]

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Abstract

We evaluate the desirability of having an elastic currency generated by a lender of last resort that prints money and lends it to banks in distress. When banks cannot borrow, the economy has a unique equilibrium that is not Pareto optimal. The introduction of unlimited borrowing at a zero nominal interest rate generates a steady state equilibrium that is Pareto optimal. However, this policy is destabilizing in the sense that it also introduces a continuum of non-optimal in‡ationary equilibria. We explore two alternate policies aimed at eliminating such monetary instability while preserving the steady-state bene…ts of an elastic currency. If the lender of last resort imposes an upper bound on borrowing that is low enough, no in‡ationary equilibria can arise. For some (but not all) economies, the unique equilibrium under this policy is Pareto optimal. If the lender of last resort instead charges a zero real interest rate, no in‡ationary equilibria can arise. The unique equilibrium in this case is always Pareto optimal.

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1. Introduction

Recent developments in a number of countries have renewed interest in the role of a lender of last resort. According to Fischer [7, p. 86] , "there is considerable agreement on the need for a domestic lender of last resort," even though there is some disagreement about exactly what this lender should do. However, several recent papers have identified the lender of last resort as a cause of excess volatility in emerging economies' financial markets and of the currency crises that have plagued many of these economies in the 1990s.[1] In response to these crises, proposals have been made in a number of countries to either establish a currency board or abolish the national currency altogether and adopt some other country's currency as legal tender (this second arrangement is often called *dollarization*). While adopting such policies may be successful in eradicating excess volatility stemming from speculation against a domestic currency, they clearly do not come without cost. In particular, both of these arrangements severely limit the ability of the central bank to act as a lender of last resort. In light of these proposals, it is important to understand the implications (both benefits and costs) of having a lender of last resort that is able to freely print money and lend to the banking system.

One of the important roles of a lender of last resort is the provision of an elastic currency, that is, the adjusting of the money supply in response to transitory changes in liquidity demand. This role was important enough to merit high billing in the act establishing the Federal Reserve System in the United States, "An act to provide for the establishment of Federal Reserve Banks, to furnish an elastic currency, ... and for other purposes." Beginning with Sargent and Wallace [17] , several papers have examined the effects of having an elastic currency supply.[2] These papers focus on stationary equilibria and show how an elastic currency promotes a more efficient allocation of resources in these equilibria. In the present paper, we show that when nonstationary equilibria are considered, the picture can change dramatically. We build on the model of Champ, Smith, and Williamson [4] , where aggregate liquidity shocks create a role for an elastic currency. In that paper, the money supply is made elastic through the issue of private banknotes. We show how having a lender of last resort that prints money and lends freely at a zero nominal interest rate generates the same result: it allows the economy to completely overcome the liquidity shocks and makes the

[1] See, for example, Chang and Velasco [5] , Mishkin [13] , and Fischer [7] .

[2] Among them are Champ, Smith, and Williamson [4] , Williamson [24] , and Freeman [8] .

stationary equilibrium Pareto optimal. However, we also show that there is a continuum of non-optimal inflationary equilibria under this regime. Hence, while having an unrestricted lender of last resort allows the economy to posses an efficient equilibrium allocation, it also opens the door to currency instability.[3]

Having identified the lender of last resort as a potential source of instability, we ask the following question: What measures could be implemented to eliminate the inefficient equilibria associated with unlimited, zero-nominal-rate lending, while retaining the benefits of such lending? We show that in some cases this may be achieved by placing a sufficiently low ceiling on the real amount banks can borrow, and that it always can be achieved by instead fixing the *real* interest rate on loans at zero.

The model is a pure exchange, two-period-lived overlapping generations economy, where some agents are lenders and others are borrowers. There is a store-of-value role for money. Agents are assigned to either of two locations at birth, and in each period a fraction of lenders is forced to move to the other location. Limited communication prevents claims on specific agents from being traded across locations and therefore only money has value in exchange after relocation. As in Townsend [21] , Mitsui and Watanabe [14] , and Hornstein and Krusell [11] , this generates a transactions role for currency and allows equilibria where money is dominated in rate of return by other assets. In this set-up, stochastic relocations act like the portfolio preference shocks commonly employed in the literature on bank runs,[4] and banks arise to insure consumers against such uncertainty. These banks write deposit contracts, hold reserves, and provide intermediation between borrowers and lenders.

In this framework, we obtain the following results. In the absence of a lender of last resort, the economy has a unique equilibrium. This equilibrium is stationary, with a constant price level and with banks holding the same fraction of their portfolio in the form of reserves at all times. There is a critical value of the relocation shock below which these precautionary reserves suffice to fully cover the demand for liquidity while equalizing the return on deposits for all agents. However, for realizations of the relocation shock above this critical value, banks face a "liquidity crisis." In this case, high liquidity demand leads to the complete exhaustion of banks' cash reserves and,

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3 See also Smith and Weber [20] , which uses a related environment to show how having an elastic currency generated by unrestricted private banknote issue can lead to even more severe indeterminacies.

4 See, for example, Diamond and Dybvig [6] , Jacklin [12] , Wallace [23] , and Peck and Shell [16] .

since other bank assets are illiquid, drives a wedge between the returns earned by depositors who are subject to the relocation shock and those who are not. Since the aggregate resources of the economy are non-stochastic, this allocation is clearly not Pareto efficient. Inflation is inconsistent with equilibrium in this setting because stochastic relocation generates a strong demand for cash reserves, even when the rate of return to holding money is low. When the money supply is constant, a sustained inflation would cause the real stock of money to go to zero and would thereby lead to an excess demand for money; in this way, inflation would preclude market clearing.

If instead the lender of last resort opens a discount window and lends freely at a zero nominal interest rate, the set of equilibria is substantially different. In this case the steady state equilibrium is Pareto optimal. Compared to the equilibrium for the benchmark case, banks hold a lower fraction of their portfolio as real balances. They obtain a discount window loan whenever reserves are insufficient to cover the demand for liquidity. By doing so they are able to fully insure agents against the random liquidity shocks: relocated and non-relocated agents earn the same return in all states of the world. However, in addition to the Pareto optimal stationary equilibrium, the economy also has a continuum of inflationary equilibria, none of which is Pareto optimal. With a lender of last resort, the effective money supply (reserves plus short-term credit) is no longer fixed. The entire point of having such a lender in this setting is to make the money supply elastic so that it responds to the stochastic movements in money demand. If there is a sustained inflation, the real value of the stock of reserves must go to zero, as before. In this case, however, the lender of last resort promises to make good on any reserve shortages through discount window loans. Hence, the availability of credit removes the strong demand for cash reserves and thereby allows inflation as an equilibrium outcome.

This result leads us to explore whether alternate discount window policies would allow the economy to preserve the desirable features of having lender-of-last-resort services without permitting inflationary equilibria. A seemingly natural constraint would be to place an upper bound on the amount of money that an individual bank can borrow. Ideally, this cap would be high enough that it never binds in the steady state (preserving the Pareto optimality of this equilibrium), but low enough that it eliminates all nonstationary equilibria. We show that whether or not this is possible depends on the distribution of the aggregate liquidity shock. We then study an economy where discount window loans have a zero *real* interest rate. In this case, inflationary equilibria are ruled out regardless of the distribution of the liquidity shock and the steady state is always Pareto optimal.

The remainder of the paper proceeds as follows. The next section lays out the basic elements of the Champ, Smith, and Williamson [4] model. Section 3 describes equilibrium without a lender of last resort, while Section 4 presents the case of unrestricted borrowing at a zero nominal interest rate. Section 5 describes the behavior of an economy where banks face an upper bound on the amount they can borrow, while Section 6 looks at a policy of fixing the real interest rate on liquidity loans. Some concluding comments are offered in Section 7.

2. The Basic Model

In this section we describe those elements of the model that are independent of the type of lender-of-last-resort services that are available to banks. The sections that follow then tailor the model to the specific policy regimes we consider.

2.1 The Environment

We begin with the pure-exchange monetary economy developed by Champ, Smith, and Williamson [4] . The economy consists of an infinite sequence of two-period lived, overlapping generations, plus an initial old generation. There is a single, perishable consumption good. At each date $t = 0, 1, \ldots$, a continuum of agents with unit mass is born at each of two identical locations. Half of these agents are "lenders" and the remaining half are "borrowers." The former have endowments $(\omega_1, \omega_2) = (x, 0)$, while the latter's endowment vector is $(\omega_1, \omega_2) = (0, y)$.[5] All consumers have $\mathbb{R}^2_{++}$ as their consumption set and have preferences given by $u(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$. We assume that $\beta x > y$ holds, which implies that this is a "Samuelson case" economy (see Gale [9]) and hence there is a role for money as a store of value. At $t = 0$ there is a continuum of old agents with unit mass in each location. Each of these agents is endowed with $M > 0$ units of fiat money, which we will refer to as "base money." The stock of base money is constant over time.

In addition to the store of value role for money, spatial separation and limited communication generate a transactions role for money in a way reminiscent of Townsend [21] , Mitsui and Watanabe

[5] The fraction of the population in each group is not important; one-half is chosen arbitrarily. All that matters is the total endowment of each group.

[14] , and Hornstein and Krusell [11] . This allows money to be dominated in rate of return by other assets. The timing of events is as follows. At the beginning of each period, all agents receive their endowments. At this point, agents cannot move between or communicate across locations. Goods can never be transported between locations. Hence, goods and asset transactions occur autarkically within each location. Young lenders can trade with old agents and can deposit resources in a bank. The bank can also trade with old agents in order to achieve the desired allocation of cash in their portfolio. Following this, young borrowers contact a bank and obtain a loan. (Note that borrowers and lenders never directly meet – all transactions are intermediated.) At this point, all agents consume. Next, a fraction π_t of young lenders in each location is notified that they will be moved to the other location. Limited communication prevents the cross-location exchange of privately issued liabilities. Currency, on the other hand, is universally recognizable and non-counterfeitable, and is therefore accepted in inter-location exchange. Movers are able to contact their bank and withdraw currency. Immediately afterwards, the movers are relocated and the next period begins. Agents now receive their old-age endowments, and borrowers use part of this endowment to repay their loans. With this revenue, banks make repayments to lenders who did not move. Lenders who did move use the currency they received from the bank to buy consumption in their new location from either young lenders or banks. At this point all old agents consume and end their lifecycle. Notice that the old-age consumption of a mover will always be equal to the real value of the money that she takes with her to the new location.[6]

The relocation probability π_t is a random variable in each period that gives the size of the aggregate liquidity shock; high values of π_t correspond to high liquidity demand. It has support $[0,1)$ and is drawn from the twice continuously differentiable, strictly increasing distribution function F with associated density function f. It is independently and identically distributed over time.

2.2 Consumers

Borrowers, who never move, face a gross market interest rate of R_t. They choose their quantity of borrowing ℓ_t to solve the problem

$$\max_{\ell_t} \ln(\ell_t) + \beta \ln(y - R_t \ell_t).$$

[6] Since the consumption set is $\mathbb{R}^2_{++}$, this implies that money must have positive value in equilibrium.

The solution to this problem is given by

$$\ell_t = \frac{y}{(1+\beta)\,R_t}. \tag{1}$$

Lenders face a more complicated problem. Given that they are confronted with random relocation, they deposit all of their savings in a bank and receive a return that depends on both whether or not they move and what fraction of all young lenders move.[7] Specifically, they are promised a real return $r_t(\pi)$ if they do not move and $r_t^m(\pi)$ if they do move. Lenders then choose the amount they save and deposit d_t to maximize expected utility, that is, to solve

$$\max_{d_t} \ln(x - d_t) + \beta \int_0^1 \pi \ln[r_t^m(\pi)\, d_t]\, f(\pi)\, d\pi + \beta \int_0^1 (1-\pi) \ln[r_t(\pi)\, d_t]\, f(\pi)\, d\pi.$$

The solution to this problem sets

$$d_t = d = \frac{\beta}{1+\beta} x. \tag{2}$$

The fact that saving is independent of the distribution of the rates of return clearly depends on the assumptions of log utility and no old-age income for lenders, which imply that the income and substitution effects of a change in the rate of return exactly offset each other.

2.3 Banks

Banks take deposits, make loans, hold reserves, and announce return schedules.[8] Any borrower can establish a bank and banks behave competitively in the sense that they take the real return on assets as given. On the deposit side, banks are assumed to behave as Nash competitors, which leads them to choose deposit returns to maximize the expected utility of young lenders. The constraints that banks face in this maximization problem depend on what lender-of-last-resort services are available to them.

Below we consider four different scenarios. First, as a benchmark case, we consider a world without a lender of last resort. We then turn our attention to the economy with a lender of last resort that provides unlimited discount window funds at a zero nominal interest rate. Next, we examine

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7 An individual's relocation status is assumed to be public information. Since in equilibrium no agent ever has an incentive to misreport her status, this seems innocuous.

8 Banks make only one type of loan, and these loans are always repaid. Thus we are abstracting from the problems of moral hazard and "excessively risky" behavior sometimes associated with the presence of a lender of last resort.

the case where banks face an upper bound on the real amount they can borrow. Finally, we analyze an economy with a lender of last resort that charges a zero real interest rate.

3. No Lender of Last Resort

In this section we discuss equilibrium for an economy in which banks are unable to borrow from anyone other than lenders. We begin by describing the bank's problem for this benchmark case, which is very similar to the bank's problem in the inelastic currency regime in Champ, Smith, and Williamson [4] . We then discuss equilibrium conditions and prove that equilibrium is unique under this policy.

3.1 The Bank's Problem

A young lender deposits her entire savings d with a bank. Per young depositor, the bank acquires an amount z_t of real balances, and makes loans with a real value $d - z_t$. The bank faces two constraints with respect to the return it promises to movers r_t^m and the return it promises to non-movers r_t. First, relocated agents, of which there are π_t, must be given currency, since that is the only asset which will allow these agents to consume at time $t+1$ in their new location. This is accomplished using a fraction $\alpha_t(\pi)$ of the bank's holdings of cash reserves. Hence, letting p_t denote the general price level at time t,[9] the return to holding money between time t and $t+1$ is given by $\frac{p_t}{p_{t+1}}$ and

$$\pi d r_t^m(\pi) \leq \alpha_t(\pi) z_t \frac{p_t}{p_{t+1}}$$

must hold. If we denote by $\gamma_t \equiv \frac{z_t}{d}$ the ratio of reserves to deposits, then we can rewrite this constraint as

$$\pi r_t^m(\pi) \leq \alpha_t(\pi) \gamma_t \frac{p_t}{p_{t+1}}. \tag{3}$$

Second, real payments to non-movers, which occur at time $t+1$, cannot exceed the value of the bank's remaining portfolio – remaining reserves plus loan repayments. Since loans earn the gross

[9] That is, p_t is the price of consumption in units of currency. Some authors (such as Wallace [22] and Balasko and Shell [3]) work instead with the inverse of p_t; the price of money in units of consumption, because it better handles situations where money has no value. In our model, the physical environment combined with the assumed consumption sets precludes equilibria with an infinite price level, and hence the two ways of defining the price system are equivalent.

real rate of return R_t, this constraint can be written as

$$(1-\pi)\, d r_t(\pi) \le (1-\alpha_t(\pi))\, z_t \frac{p_t}{p_{t+1}} + (d - z_t) R_t$$

or

$$(1-\pi)\, r_t(\pi) \le (1-\alpha_t(\pi))\, \gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t. \tag{4}$$

Of course, $0 \le \gamma_t \le 1$ and $0 \le \alpha_t(\pi) \le 1$ must hold.

Because banks behave as Nash competitors and there is free entry, banks will maximize young lenders' utility, taking deposit demand d as given. Given (2), the bank's problem is then to choose $r(\pi)$ and $r^m(\pi)$ to maximize

$$\ln\left(\frac{x}{1+\beta}\right) + \beta \int_0^1 \left\{ \pi \ln\left[r_t^m(\pi) \frac{\beta x}{1+\beta}\right] + (1-\pi) \ln\left[r_t(\pi) \frac{\beta x}{1+\beta}\right]\right\} f(\pi)\, d\pi \tag{5}$$

subject to the constraints (3) and (4), which will hold with equality at an optimum. Substituting in these constraints and dropping the constant terms yields the problem

$$\max_{\alpha_t(\pi);\, \gamma_t} \int_0^1 \left\{ \pi \ln[\alpha(\pi)\gamma_t] + (1-\pi) \ln\left[(1-\alpha(\pi))\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right]\right\} f(\pi)\, d\pi \tag{6}$$

subject to

$$0 \le \gamma_t \le 1$$
$$0 \le \alpha_t(\pi) \le 1.$$

The function α_t, which is the fraction of bank reserves paid out to movers, is chosen after the realization of π, while the function γ_t, the fraction of reserves in the bank's asset portfolio, is chosen before the realization of π. Hence we can first determine the optimal value of α_t for fixed values of γ_t and π. That is, we can choose α_t to solve

$$\max_{0 \le \alpha_t \le 1} \pi \ln[\alpha_t \gamma_t] + (1-\pi) \ln\left[(1-\alpha_t)\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right].$$

The solution to this problem sets

$$\alpha_t(\pi) = \begin{cases} \pi\left(1 + \frac{1-\gamma_t}{\gamma_t} R_t \frac{p_{t+1}}{p_t}\right) \\ 1 \end{cases} \text{for } \pi \in \begin{cases} [0, \pi^*) \\ [\pi^*, 1) \end{cases};$$

where we have

$$\pi^* = \frac{\gamma_t \frac{p_t}{p_{t+1}}}{\gamma_t \frac{p_t}{p_{t+1}} + (1 - \gamma_t) R_t}. \tag{7}$$

For realizations of the relocation shock below the critical value π^*, the bank pays out only a fraction of its reserves to movers, and both movers and non-movers receive the same return. When the realization of the relocation shock is greater than π^*, the bank faces a "liquidity crisis." It pays out all its cash reserves to movers, while repayments to non-movers are drawn from loan repayments only. In a crisis, the bank cannot equalize the returns of movers and non-movers; movers must receive a lower return.

It remains to determine the optimal value of γ_t. To do so, we substitute the optimal value of α_t into the bank's objective function so that the only remaining choice variable is γ_t. Doing so yields the problem

$$\max_{0 \le \gamma_t \le 1} \int_0^{\pi^*} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1 - \gamma_t) R_t\right] f(\pi)\, d\pi + \int_{\pi^*}^1 \left(\pi \ln\left[\gamma_t \frac{p_t}{p_{t+1}}\right] + (1 - \pi) \ln\left[(1 - \gamma_t) R_t\right]\right) f(\pi)\, d\pi.$$

This formulation of the problem makes it clear that the return earned by both movers and non-movers will be the same when π is less than π^*, but will in general be different when π is greater than π^*. The first-order condition for this problem is

$$\frac{R_t - \frac{p_t}{p_{t+1}}}{\gamma_t \frac{p_t}{p_{t+1}} + (1 - \gamma_t) R_t} F(\pi^*) = \frac{1}{\gamma_t} \int_{\pi^*}^1 \pi f(\pi)\, d\pi - \frac{1}{1 - \gamma_t} \int_{\pi^*}^1 (1 - \pi) f(\pi)\, d\pi.$$

This can be reduced to[10]

$$\gamma_t = 1 - \int_{\pi^*}^1 F(\pi)\, d\pi. \tag{8}$$

This implicitly defines the solution to the bank's problem when no lender-of-last-resort services are provided. The optimal γ_t results from the trade-off between two forces. First, the return on cash balances is lower than the return on loans, and therefore the bank would like to economize on reserve holdings. On the other hand, the bank strives to provide insurance by equalizing the returns given to movers and non-movers. To be able to do so, it must hold sufficient cash balances. At the margin, the welfare gains from equalizing the returns to movers and non-movers must exactly

[10] The intermediate steps are provided in Appendix A.

offset the cost implied by the return dominance of loans over cash reserves.

3.2 Equilibrium

An equilibrium of this economy is characterized by the market clearing conditions for real balances and loans. Because the supply of real balances is equal to $\frac{M}{p_t}$ and the demand for real balances is given by $\gamma_t d$, market clearing for real balances and (2) require that we have

$$\frac{M}{p_t} = \gamma_t \frac{\beta}{1+\beta} x.$$

Similarly, the demand for loans is given in (1), while the supply of loans is given by $(1 - \gamma_t) d.$ Together these yield the market clearing condition for loans,

$$\frac{y}{(1+\beta) R_t} = (1 - \gamma_t) \frac{\beta}{1+\beta} x.$$

These equations imply that in equilibrium we must have both

$$\gamma_t \frac{p_t}{p_{t+1}} = \gamma_{t+1} \tag{9}$$

and

$$R_t (1 - \gamma_t) = \frac{y}{\beta x}. \tag{10}$$

Substituting (9) and (10) into the expression for π^* in (7) yields

$$\pi^* = \frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{y}{\beta x}};$$

which we can substitute into (8) to obtain the difference equation

$$\gamma_t = 1 - \int_{\frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{y}{\beta x}}}^{1} F(\pi)\, d\pi. \tag{11}$$

This implicitly defines the law of motion for $\gamma_t.$ The properties of this law of motion give us the following proposition.

Proposition 1 When there is no lender of last resort, the economy has a unique equilibrium. This equilibrium is stationary with $\gamma_t = \gamma_a$ for all t, and $\max\left\{E(\pi); 1 - \frac{y}{\beta x}\right\} < \gamma_a < 1.$

The proof of this proposition is presented in Appendix B and is illustrated in Fig. 1. The law

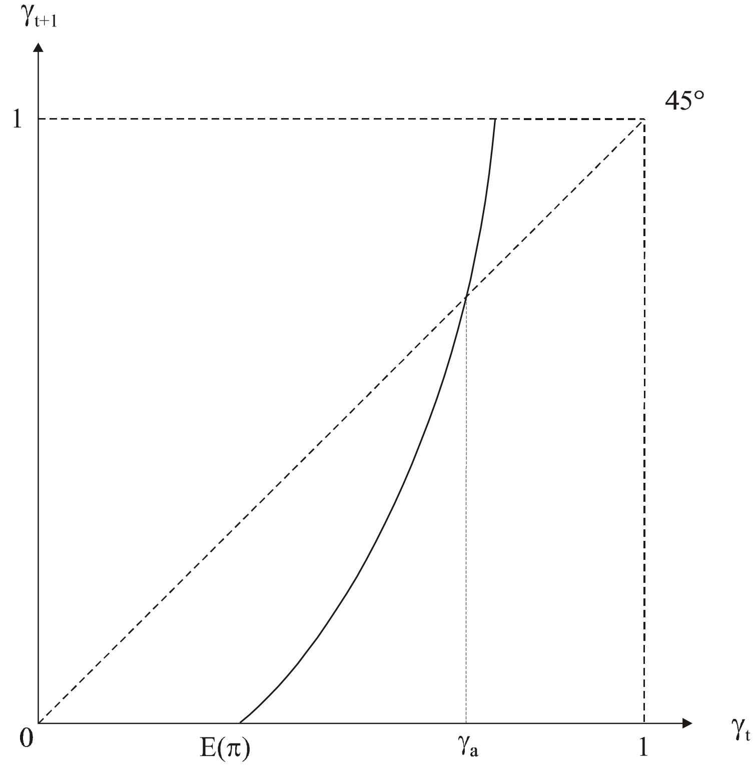

Figure 1: No Lender of Last Resort

of motion implicitly defined in (11) crosses the forty-five degree line exactly once, and this steady state is the only equilibrium of the economy. The absence of inflationary equilibria follows from the strong demand for currency generated by logarithmic utility and the fact that relocated agents need money to consume. In this model, there is a positive lower bound on the demand for real money balances. As the rate of return to holding money goes to zero, real money demand approaches E [¼] d (this follows from (8) using (7)). Hence an inflationary trajectory, along which $°_t$ would go to zero, cannot be consistent with market clearing. Therefore, unlike the standard Samuelson-case economy discussed in Gale [9] , this model cannot have inflationary equilibria when the money supply is constant. The steady state is the unique equilibrium.

Notice, however, that this equilibrium is not Pareto efficient. There are states of the world in which the consumptions of relocated and non-relocated lenders are different, even though there is no uncertainty about the aggregate resources of the economy. The problem is that banks must choose their reserve holdings before money demand is realized. If the bank could adjust these holdings once demand is known by, say, borrowing from a lender of last resort when money demand is high, it seems possible that a more efficient outcome could be achieved. We study various such lending regimes in the remaining sections.

4. Lending at a Zero Nominal Interest Rate

In this section we analyze the regime in which the lender of last resort opens a discount window and makes one-period loans of currency at a zero nominal interest rate in any quantity that banks desire. Note that this policy is always feasible, in that it requires no real resources from the lender of last resort. After the realization of π, a bank determines the real amount $b \geq 0$ that it would like to borrow at time t – which will depend on the realization of π – and obtains bp_t dollars from the discount window. In the following period, the bank must return these dollars to the window and they are destroyed. In this way, the stock of beginning-of-period base money remains fixed.

4.1 The Bank's Problem

Defining $\delta_t \equiv \frac{b_t}{d}$ to be real borrowing per unit of deposits, the bank's constraints become

$$\pi r_t^m(\pi) = \alpha_t(\pi)\gamma_t \frac{p_t}{p_{t+1}} + \delta_t(\pi)\frac{p_t}{p_{t+1}} \tag{12}$$

and

$$(1-\pi) r_t(\pi) = (1-\alpha_t(\pi))\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t - \delta_t(\pi)\frac{p_t}{p_{t+1}}. \tag{13}$$

The introduction of zero nominal interest rate borrowing allows us to collapse these into a single constraint,

$$\pi r_t^m(\pi) + (1-\pi) r_t(\pi) = \gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t. \tag{14}$$

The bank chooses the two returns to maximize (5) subject to this constraint. The solution to this problem has

$$r_t^m(\pi) = r_t(\pi) \quad \text{for all } \pi;$$

that is, depositors receive perfect insurance against the relocation shock. One way the bank could generate these returns is by setting

$$\alpha_t(\pi) = \begin{Bmatrix} \pi\left(1 + \frac{1-\gamma_t}{\gamma_t} R_t \frac{p_{t+1}}{p_t}\right) \\ 1 \end{Bmatrix} \quad \text{and} \quad \delta_t(\pi) = \begin{Bmatrix} 0 \\ \gamma_t \left[\pi\left(1 + \frac{1-\gamma_t}{\gamma_t} R_t \frac{p_{t+1}}{p_t}\right) - 1\right] \end{Bmatrix}$$

$$\text{for } \pi \in \begin{Bmatrix} [0; \pi^*) \\ [\pi^*; 1) \end{Bmatrix};$$

where π^* continues to be given by (7). For realizations of the relocation shock below the critical value π^*, the bank pays out only a fraction of its reserves to movers and, under this plan, does not obtain a discount window loan. When the relocation shock is larger than π^*, the bank does obtain a loan from the discount window and pays out this loan plus reserves to movers. At the beginning of next period, non-movers are paid what remains after the bank has repaid the discount window loan. Note, however, that the bank could also borrow money when π is below π^*; give this money to movers, and use cash reserves to repay the loan next period. With a zero nominal interest rate, an individual bank's demand for loans from the discount window is not uniquely determined. What is determined, however, is the real value of the money given to movers. This is always chosen to equate the returns to movers and non-movers.

Since movers and non-movers receive the same return, both must receive the average return on the bank's portfolio, which is the right-hand-side of (14). In order to maximize this return, the optimal choice of reserve-deposit ratio γ_t must be given by

$$\gamma_t = \begin{Bmatrix} 0 \\ \in [0; 1] \\ 1 \end{Bmatrix} \quad \text{as} \quad R_t \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \frac{p_t}{p_{t+1}}. \tag{15}$$

4.2 Equilibrium

The market-clearing equations are the same as in the previous section, and hence (9) and (10) continue to hold. In equilibrium we cannot have $\gamma_t = 0$, because this would imply that the price level is infinite, which in turn would imply that movers would have zero old-age consumption. We cannot have $\gamma_t = 1$ either, since then borrowers would have zero young-period consumption.

Therefore, in equilibrium the pricing relationship

$$R_t = \frac{p_t}{p_{t+1}} \tag{16}$$

must obtain. After substituting (16) into (9) and (10), the market clearing conditions simplify to the law of motion for γ,

$$\gamma_{t+1} = \frac{y}{x}\frac{\gamma_t}{1-\gamma_t}. \tag{17}$$

The properties of this law of motion give us the following proposition.

Proposition 2 When the lender of last resort offers unrestricted loans with a zero nominal interest rate, the economy has a continuum of equilibria. There is a Pareto optimal stationary equilibrium for $\gamma_0 = 1 - \frac{y}{x} = \gamma_b$, and a continuum of non-optimal, inflationary equilibrium paths for $\gamma_0 \in (0; \gamma_b)$.

The proof of Proposition 2 is straightforward and therefore omitted. The results of this proposition are illustrated in Fig. 2. The striking feature of the new law of motion is that it permits inflationary

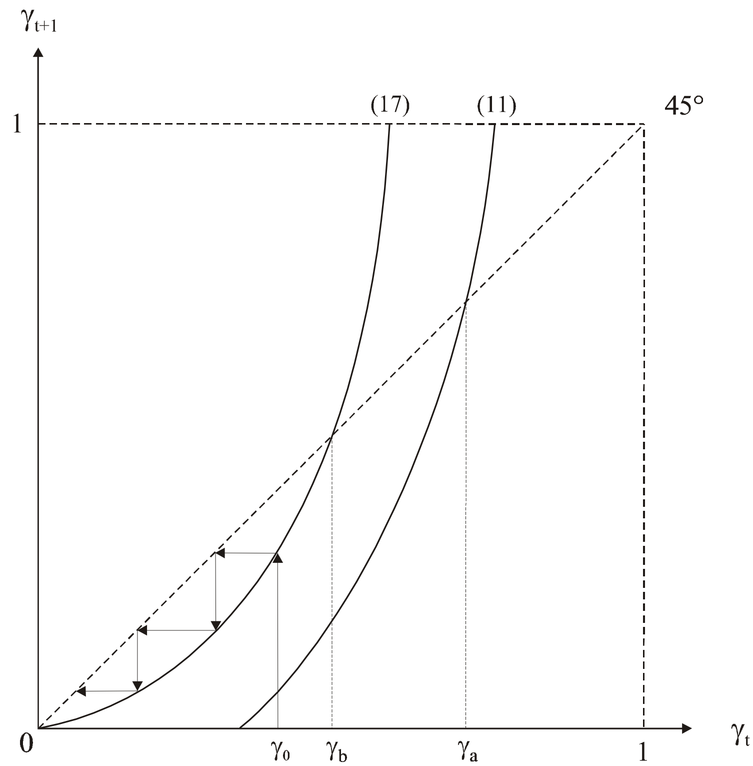

Figure 2: A Zero Nominal Interest Rate

equilibria, where γ_t asymptotically approaches zero. It is clear from (16) that the equilibrium nominal interest rate is always zero here, and that therefore the lender of last resort is charging exactly the market rate on loans. Because of this, there is no “penalty” if a bank’s reserve holdings turn out to be too low. The bank can simply borrow cash at the same interest rate that it is earning on its real lending. This is what generates the indeterminacy of the bank’s portfolio decision (15), and it implies that there is no lower bound on the demand for reserves. As a result, a sustained inflation, where aggregate reserve holdings must go to zero, is consistent with equilibrium in this case. As reserve holdings decrease, borrowing from the discount window increases. In this way, the lender of last resort responds to inflation by increasing short-term credit, which in turn makes inflation consistent with equilibrium.

In the steady state, the provision of zero nominal interest rate loans allows the economy to completely overcome the stochastic relocation friction. Since banks can now borrow money when the demand for it is high, they no longer hold precautionary reserves and therefore the steady state reserve-deposit ratio is smaller than in the case without a lender of last resort. In fact, the law of motion (17) is identical to the one that would obtain if there were no relocations in this economy. It is well known that the steady state is Pareto optimal in this case, but that the inflationary equilibria are not.[11]

In summary, the introduction of this type of lending generates a Pareto optimal equilibrium. However, it also generates a continuum of inflationary equilibria that are not Pareto efficient. Is it better to have a lender of last resort or not? There are no clear criteria for answering such a question, since it involves comparing the *sets* of equilibria generated by two different policies. Rather than address it directly, we take the approach used in Shell [18] , Grandmont [10] , Woodford [25] , and Smith [19] (among others). We ask if it is possible to design a policy that captures the benefits of providing lender-of-last-resort services without introducing inflationary equilibria. We study two policies, the first of which restricts the amount of borrowing that can be undertaken and the second of which involves fixing the real interest rate on discount window loans.

5. An Upper Bound on Borrowing

The analysis above shows that the ability to borrow at a discount window undermines the incentive

11 This follows from Proposition 5.6 in Balasko and Shell [2] . See also p.838 in Champ, Smith and Williamson [4] .

for banks to hold reserves and that this is the source of the resulting inflationary equilibria. In this section we suppose that the lender of last resort places an upper bound on the real amount that a bank can borrow.[12] We show that if this bound is low enough, it will restore the bank's demand for cash reserves and thereby eliminate the inflationary equilibria. We ask if the bound can at the same time be high enough to never bind in the steady state. We show that whether or not this is the case depends on the distribution of the liquidity shocks.

5.1 The Bank's Problem

We use $c \in (0,\infty)$ to denote the real amount that a bank can borrow per unit of deposits that it holds. The bank continues to face the constraints (12) and (13). Substituting these constraints into the bank's objective function (5), dropping the constant terms, and taking into account the upper bound on borrowing yields the problem

$$\max_{\alpha_t(\pi);\, \delta_t(\pi);\, \gamma_t} \int_0^1 \pi \ln\left[\alpha(\pi)\gamma_t + \delta_t(\pi)\right] f(\pi)\, d\pi + \int_0^1 (1-\pi) \ln\left[(1-\alpha(\pi))\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t - \delta_t(\pi)\frac{p_t}{p_{t+1}}\right] f(\pi)\, d\pi$$

subject to

$$0 \le \gamma_t \le 1$$
$$0 \le \alpha_t(\pi) \le 1$$
$$0 \le \delta_t \le c.$$

As before, we can first determine the optimal values of α_t and δ_t for given values of γ_t and π. Clearly, the borrowing constraint can only be binding in some states if its value is smaller than the value of the loan a bank would take for $\pi = 1$ in the absence of the constraint. Hence, the problem of choosing α_t and δ_t here differs from the one in Section 4 only if reserve holdings are low enough that

$$\gamma_t < 1 - \frac{p_t}{p_{t+1}R_t}c \equiv \hat{\gamma}$$

[12] An upper bound on the amount of *nominal* borrowing would always be effective in eliminating inflationary equilibria, but there may be credibility issues with such a bound. Constraints on the real amount of borrowing may be easier to commit to; as an example one might think of a dollarized economy where the central bank has accumulated a stock of dollars and can lend from this stock but cannot print more.

holds. We begin with this case. For low values of π, the bank's optimal amount of borrowing is not uniquely determined, as in the previous section. However, the solution to the problem again involves equating the returns of movers and non-movers whenever this is possible. One way of doing this is to set

$$\alpha_t(\pi) = \left\{ \begin{array}{c} \pi\left(1 + \frac{1-\gamma_t}{\gamma_t} R_t \frac{p_{t+1}}{p_t}\right) \\ 1 \\ 1 \end{array} \right\} \text{ and } \delta_t(\pi) = \left\{ \begin{array}{c} 0 \\ \gamma_t \left[\pi\left(1 + \frac{1-\gamma_t}{\gamma_t} R_t \frac{p_{t+1}}{p_t}\right) - 1\right] \\ c \end{array} \right\}$$

$$\text{for } \pi \in \left\{ \begin{array}{c} [0; \pi^*) \\ [\pi^*; \pi^{**}) \\ [\pi^{**}; 1) \end{array} \right\};$$

where π^* continues to be given by (7) and π^{**} is given by

$$\pi^{**} = \frac{(\gamma_t + c)\frac{p_t}{p_{t+1}}}{\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t}. \tag{18}$$

Note that this expression for π^{**} is less than unity if and only if $\gamma_t < \hat{\gamma}$ holds. If $\gamma_t \geq \hat{\gamma}$ holds, the bank can equalize returns for movers and non-movers for all values of π, as in the previous section.

We can now determine the bank's optimal portfolio in the presence of borrowing constraint. To do so, we substitute the information above into the bank's objective function. This yields the problem

$$\max_{0 \leq \gamma_t \leq 1} \int_0^{\pi^*} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right] f(\pi)\, d\pi + \int_{\pi^*}^{\pi^{**}} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right] f(\pi)\, d\pi +$$
$$\int_{\pi^{**}}^1 \left(\pi \ln[\gamma_t + c] + (1-\pi) \ln\left[(1-\gamma_t) R_t - c \frac{p_t}{p_{t+1}}\right]\right) f(\pi)\, d\pi;$$

which can be rewritten as

$$\max_{0 \leq \gamma_t \leq 1} \int_0^{\pi^{**}} \ln\left[\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t\right] f(\pi)\, d\pi +$$
$$\int_{\pi^{**}}^1 \left(\pi \ln[\gamma_t + c] + (1-\pi) \ln\left[(1-\gamma_t) R_t - c \frac{p_t}{p_{t+1}}\right]\right) f(\pi)\, d\pi.$$

Here we see that the returns earned by movers and non-movers will be the same when π is less than

π^{**}, but will be different when π is greater than π^{**}. The first-order condition for this problem is

$$\frac{\frac{p_t}{p_{t+1}} - R_t}{\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t}\int_0^{\pi^{**}} f(\pi)d\pi + \frac{1}{\gamma_t + c}\int_{\pi^{**}}^1 \pi f(\pi)d\pi = \frac{R_t}{(1-\gamma_t)R_t - c\frac{p_t}{p_{t+1}}}\int_{\pi^{**}}^1 (1-\pi)f(\pi)d\pi:$$

which can be reduced to[13]

$$\gamma_t = 1 - \frac{1-\pi^{**}+c}{1-\pi^{**}}\int_{\pi^{**}}^1 F(\pi)d\pi: \tag{19}$$

This equation implicitly defines the optimal portfolio allocation when its solution satisfies $\gamma_t < \bar{\gamma}$: Otherwise, the optimal allocation resembles that in the previous section: the bank is indifferent between any γ_t in $[\bar{\gamma}; 1]$ as long as $R_t = \frac{p_t}{p_{t+1}}$ holds.

5.2 Equilibrium

The market-clearing conditions (9) and (10) continue to hold. Substituting these equations into the expression for π^{**} in (18) yields

$$\pi^{**} = \frac{\gamma_{t+1} + \frac{\gamma_{t+1}}{\gamma_t}c}{\gamma_{t+1} + \frac{y}{\bar{x}}}:$$

Substituting this into (19), we obtain

$$\gamma_t = 1 - \frac{\frac{y}{\bar{x}} - c\left(\frac{\gamma_{t+1}}{\gamma_t} - \gamma_{t+1} - \frac{y}{\bar{x}}\right)}{\frac{y}{\bar{x}} - c\frac{\gamma_{t+1}}{\gamma_t}}\int_{\frac{\gamma_{t+1}+\frac{\gamma_{t+1}}{\gamma_t}c}{\gamma_{t+1}+\frac{y}{\bar{x}}}}^1 F(\pi)d\pi: \tag{20}$$

This implicitly defined law of motion applies when $\gamma_t < \bar{\gamma}$ holds, or when we have

$$\gamma_{t+1} < \frac{y}{c\bar{x}}\gamma_t:$$

As shown in Fig. 3, the phase plane is divided into two regions. Below the lower dashed line, the law of motion is given by (20). Above this line, it is given by (17). Both curves intersect the dashed line at $\gamma_t = 1 - c > 0$, and therefore the piecewise-defined law of motion is continuous. Whether or not the bound affects the steady state equilibrium simply depends on whether or not $\gamma_t < \bar{\gamma}$ holds when $\gamma_{t+1} = \gamma_t$: That is, c is binding in some states in the stationary equilibrium if

13 The intermediate steps are provided in Appendix C.

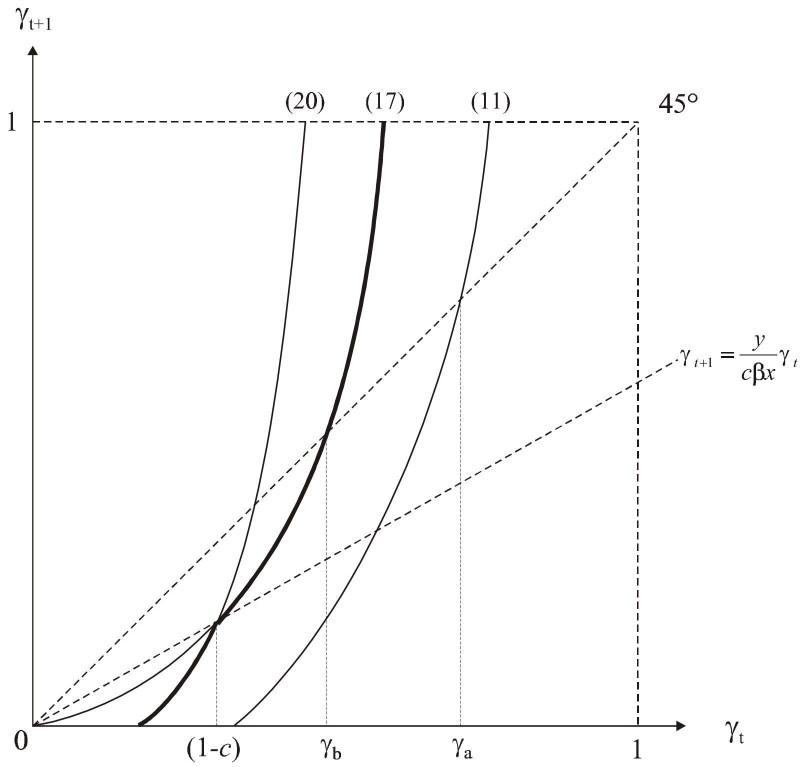

Figure 3: An Upper Bound on Borrowing

and only if we have

$$c < \frac{y}{x}.$$

Whether or not there exist inflationary equilibria is determined by (20), since this governs the law of motion near the origin. The demand for reserves is given by (19). Taking the limit as the return to holding money goes to zero, we have

$$\lim_{\frac{p_t}{p_{t+1}} \to 0} \gamma_t = E[\pi] - c(1 - E[\pi]),$$

which can be either positive or negative. If the upper bound is low enough for this to be positive, that is, if

$$c < \frac{E[\pi]}{1 - E[\pi]}$$

holds, then the demand for reserves has a positive lower bound. This case is qualitatively similar to having no lender of last resort $(c = 0)$: Demand for reserves never goes to zero, and therefore sustained inflations are not possible in equilibrium. This is the case depicted in Fig. 3, where the law of motion intersects the horizontal axis to the right of the origin.

If instead we have

$$c > \frac{E[\pi]}{1 - E[\pi]};$$

then the demand for reserves goes to zero when the return to holding money approaches some finite number. In this case the part of the law of motion given by (20) also begins at the origin, and hence the set of equilibria is qualitatively similar that when there is an unrestricted lender of last resort $(c = 1)$: There is a continuum of inflationary equilibria, none of which are Pareto optimal. Finally, if we happen to have

$$c = \frac{E[\pi]}{1 - E[\pi]};$$

the demand for reserves goes to zero only as the rate of return to holding money goes to zero. In this case there are true hyperinflationary equilibria where the inflation rate grows without bound. The following proposition formalizes this result.

Proposition 3 When the lender of last resort sets an upper bound on borrowing $c \in \left(\frac{y}{\beta x}; 1\right)$, there is a stationary equilibrium with $\gamma_t = \gamma_b$ for all t. If $c < \frac{E[\pi]}{1-E[\pi]}$ holds, this is the unique equilibrium. If instead $c \geq \frac{E[\pi]}{1-E[\pi]}$ holds, there is also a continuum of inflationary paths for $\gamma_0 \in (0; \gamma_b)$:

The proof of this proposition closely follows the reasoning given above and is therefore omitted. It is interesting to note that the condition for c to affect the steady state equilibrium and the condition for it to eliminate inflationary equilibria are unrelated. If the distribution of liquidity shocks satisfies

$$E[\pi] > \frac{y}{y + \beta x};$$

then an upper bound of this sort is an ideal policy. The cap can be chosen high enough to never bind in the steady state (making this equilibrium efficient), while still being low enough to eliminate inflationary equilibria. Note that this condition necessarily holds if the expected value of π is at least $\frac{1}{2}$; as it is for the uniform distribution. If, however, high liquidity demand is a rare event (and hence $E[\pi]$ is low), the bound required to eliminate the inflationary equilibria would be low and the stationary equilibrium would exhibit periodic crises.

6. Lending at a Zero Real Interest Rate

We now return to a situation where the discount window offers one-period loans of currency in any quantity that banks desire. However, the interest rate on these loans is now fixed in real terms (at zero) as long as the inflation rate is nonnegative. Specifically, after the realization of π, a bank determines the real amount $b \geq 0$ that it would like to borrow at time t (which will depend on the realization of π) and obtains bp_t dollars from the discount window. Next period, the bank must return bp_{t+1} dollars as long as $p_{t+1} \geq p_t$ holds, and bp_t dollars otherwise. The reason for this two-part rule is that under a deflation, the ability to borrow at a zero real interest rate would generate an arbitrage opportunity for banks (at the expense of the lender of last resort). For the announced policy to be feasible under all possible inflation rates, the nominal interest rate can never be negative. Another way of stating the policy that we study here is that it sets the nominal interest rate to $\max\left\{\frac{p_{t+1}}{p_t} - 1; 0\right\}$.[14]

In the event of an inflation, the lender of last resort earns positive profits on discount window loans under this policy. We assume that the lender of last resort then engages in purchases of goods so that the stock of beginning-of-period base money remains unchanged at M: We further assume that agents derive no utility from these purchases. If instead the revenue were rebated to banks as a state-contingent, lump-sum payment, the qualitative properties of the results would not change. Because such rebates complicate the algebra substantially, we present the simpler case here.

6.1 The Bank's Problem

Again letting $\delta_t = \frac{b_t}{d}$ denote real borrowing per unit of deposits, the bank's constraints under this arrangement are

$$\pi r_t^m(\pi) = \alpha_t(\pi)\gamma_t \frac{p_t}{p_{t+1}} + \delta_t(\pi)\frac{p_t}{p_{t+1}} \tag{21}$$

and

$$(1-\pi) r_t(\pi) = (1-\alpha_t(\pi))\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t) R_t - \min\left\{1; \frac{p_t}{p_{t+1}}\right\}\delta_t(\pi):$$

Substituting these into the bank's objective function (5) and dropping the constant terms yields the

[14] Under this policy and many others, deflation is not an equilibrium outcome. Because of this, the exact form that the policy takes in the case of a deflation is not important. We have chosen this one simply because much of the needed analsysis has already been given in Section 4.

problem

$$\max_{\alpha_t(\pi);\,\delta_t(\pi);\,\gamma_t} \int_0^1 \pi \ln\left[\alpha(\pi)\gamma_t + \delta_t(\pi)\right] f(\pi)\, d\pi + \int_0^1 (1-\pi)\ln\left[\begin{array}{c}(1-\alpha(\pi))\gamma_t\frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t \\ -\min\left\{1; \frac{p_t}{p_{t+1}}\right\}\delta_t(\pi)\end{array}\right] f(\pi)\, d\pi \tag{22}$$

subject to

$$0 \le \gamma_t \le 1$$
$$0 \le \alpha_t(\pi) \le 1$$
$$\delta_t(\pi) \ge 0.$$

We break the solution of the bank's problem into two cases. First, suppose that the price level is either constant or falling between two periods $(p_{t+1} \le p_t)$. In this case, borrowing under the stated policy is the same as borrowing at a zero nominal interest rate, and the solution to the bank's problem is the same as in section 4. Therefore the solution is characterized by (15) and $r_t^m(\pi) = r_t(\pi)$.

If instead there is inflation $(p_{t+1} > p_t)$, the bank's problem is more complex. As in the previous sections, we can first solve for the optimal values of α_t and δ_t given γ_t and π. That is, we can choose α_t and δ_t to solve

$$\max_{\alpha_t;\delta_t} \pi \ln\left[\alpha_t\gamma_t + \delta_t\right] + (1-\pi)\ln\left[(1-\alpha_t)\gamma_t\frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t - \min\left\{1; \frac{p_t}{p_{t+1}}\right\}\delta_t\right]$$

subject to

$$0 \le \alpha_t \le 1$$
$$\delta_t \ge 0.$$

The solution to this problem sets

$$\alpha_t(\pi) = \left\{\begin{array}{c} \pi\left(1 + \frac{1-\gamma_t}{\gamma_t}R_t\frac{p_{t+1}}{p_t}\right) \\ 1 \\ 1 \end{array}\right\} \text{ and } \delta_t(\pi) = \left\{\begin{array}{c} 0 \\ 0 \\ \gamma_t\left[\pi\left(1 + \frac{1-\gamma_t}{\gamma_t}R_t\right) - 1\right] \end{array}\right\}$$

$$\text{for } \pi \in \left\{\begin{array}{c} [0; \pi^*) \\ [\pi^*; \pi^{**}) \\ [\pi^{**}; 1) \end{array}\right\};$$

where π^* continues to be given by (7) and we have

$$\pi^{**} = \frac{\gamma_t}{\gamma_t + (1-\gamma_t)R_t} < 1. \qquad (23)$$

As in the previous sections, when the realization of π is low, the bank equates the returns received by movers and non-movers by giving only a fraction of its reserves to movers. When a relocation shock between π^* and π^{**} materializes, all reserves are paid out to movers, but the bank does not resort to a discount window loan. Only when the relocation shock is larger than π^{**} does the bank obtain a loan. The range of inaction $[\pi^*; \pi^{**}]$ is generated by a kink in the bank's opportunity set. Once the level of reserves is set, the bank has a certain amount of currency on hand and the return to holding that currency is $\frac{p_t}{p_{t+1}}$. The cost of acquiring additional currency, however, is unity. The increase in lenders' expected utility must be sufficiently large before the bank will undertake any borrowing at this rate.

Given this optimal schedule for α_t and δ_t, the bank chooses γ_t to solve (22). The first-order condition for this problem is

$$\frac{R_t - \frac{p_t}{p_{t+1}}}{\gamma_t \frac{p_t}{p_{t+1}} + (1-\gamma_t)R_t} F(\pi^*) + \frac{R_t - 1}{\gamma_t + (1-\gamma_t)R_t}\left[1 - F(\pi^{**})\right]$$
$$= \frac{1}{\gamma_t}\int_{\pi^*}^{\pi^{**}} \pi f(\pi)\, d\pi - \frac{1}{1-\gamma_t}\int_{\pi^*}^{\pi^{**}} (1-\pi) f(\pi)\, d\pi;$$

which can be reduced to[15]

$$\gamma_t = \pi^{**} - \int_{\pi^*}^{\pi^{**}} F(\pi)\, d\pi; \qquad (24)$$

which implicitly defines the solution to the bank's problem when there is inflation.

6.2 Equilibrium

The market-clearing equations are the same as in the benchmark case, and hence (9) and (10) continue to hold. We divide the phase plane into two regions and derive the equilibrium law of motion in each region. We begin with the region where $\gamma_{t+1} \geq \gamma_t$ holds. In this case (9) implies that $p_{t+1} = p_t$ holds and therefore we must have $R_t = 1$ for all t. This implies $\gamma_t = 1 - \frac{y}{x} = \gamma_b$ for all t, and therefore we have the same steady-state law of motion as in section 4. Notice in particular

15 The intermediate steps are provided in Appendix D.

that this implies that we have the same steady-state equilibrium as in Section 4.

Next we examine the region where $\gamma_{t+1} < \gamma_t$. From (9) it is clear that equilibria in this region would exhibit inflation. Substituting (9) and (10) into the expression for $\pi^{\star}$ in (7) and the expression for $\pi^{\star\star}$ in (23) yields

$$\pi^{\star} = \frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{y}{x}} \quad \text{and} \quad \pi^{\star\star} = \frac{\gamma_t}{\gamma_t + \frac{y}{x}}.$$

Substituting these into (24), we obtain the graph of the law of motion for γ_t that applies in this region,

$$\gamma_t = \frac{\gamma_t}{\gamma_t + \frac{y}{x}} - \int_{\frac{\gamma_{t+1}}{\gamma_{t+1}+\frac{y}{x}}}^{\frac{\gamma_t}{\gamma_t+\frac{y}{x}}} F(\pi)\, d\pi. \tag{25}$$

We can now state the following proposition.

Proposition 4 When the lender of last resort charges a zero *real* interest rate on discount window loans, the economy has a unique equilibrium. This equilibrium is stationary, with $\gamma_t = \gamma_b$ for all t, and is Pareto optimal.

The proof of Proposition 4 is presented in Appendix E and is illustrated in Fig. 4. Fixing the real interest rate rather the nominal rate is effective in eliminating inflationary equilibria because, during an inflation, the lender of last resort is charging a higher rate than the market rate on loans. Along an inflationary path, the interest rate on real loans to borrowers is falling to $\frac{y}{x} < 1$; but the cost of borrowing from the discount window is fixed at unity. Hence this policy is in line with the recommendation of Bagehot [1] that "in a crisis, the lender of last resort should lend freely, at a penalty rate." Under this policy, the lender of last resort is charging a penalty rate if and only if there is inflation. This generates a lower bound on the demand for reserves. To see why, suppose that the economy follows an inflationary trajectory. As the real stock of base money decreases, banks engage in more real lending and the rate of return to real lending falls. Imagine a situation where γ has become very close to zero, that is, where there has been sustained inflation for many periods. This implies that in practically every period, the bank will be borrowing currency at a cost of unity. At the same time, the return the bank is receiving from its real lending is close to $\frac{y}{x} < 1$. Hence, regardless of the rate of return on money, the bank would be better off holding more reserves and engaging in less lending simply because borrowing is so expensive. This means there is a lower bound on the demand for reserves, even as the rate of return to holding money goes to zero. For

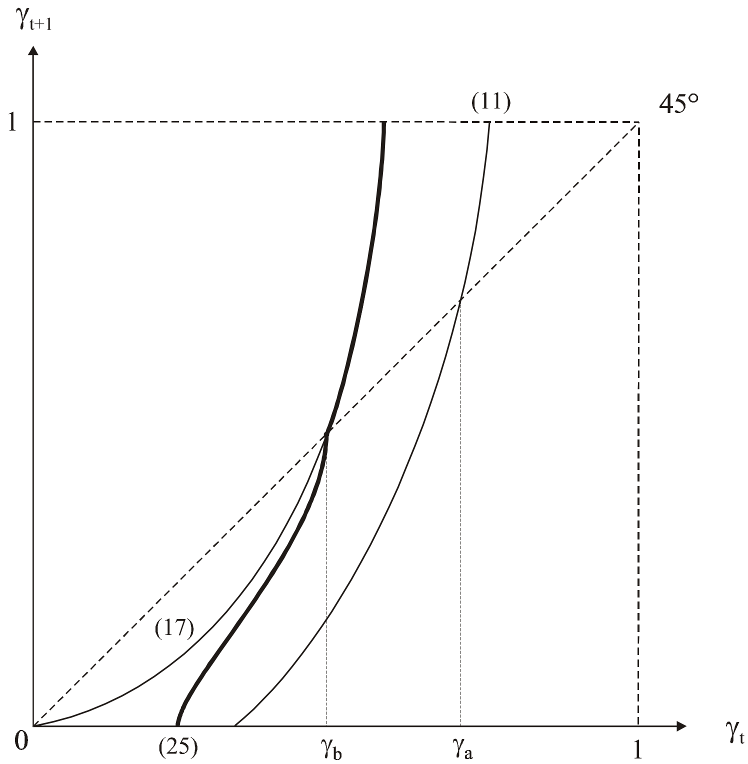

Figure 4: A Zero Real Interest Rate

this reason there cannot be a sustained inflation.

It is interesting to note that fixing the nominal interest rate at some level above zero cannot eliminate inflationary equilibria. In this situation, the inflation rate can always adjust so that the lender of last resort is charging exactly the market rate on real loans. This removes the lower bound on the demand for base money and permits inflation as an equilibrium outcome. Fixing the real interest rate works because it guarantees that the lender of last resort is charging a penalty rate for any positive rate of inflation.

7. Conclusions

We have studied a pure-exchange economy in which spatial separation, limited communication and random relocation combine to create a role for money, even when it is dominated in rate of return. Banks arise in this world to insure agents against the liquidity shocks implied by random relocation. When the money supply is constant, the economy has a unique equilibrium that is not

Pareto optimal. This equilibrium is marked by periodic crises in which high aggregate liquidity demand leads to low consumption levels for agents in need of liquidity.

When we introduce a lender of last resort providing unlimited, zero-nominal-interest-rate loans to banks in distress, the stationary equilibrium is Pareto optimal. However, there is a continuum of inflationary equilibria that are not Pareto efficient. Thus, while allowing the economy to overcome the frictions associated with stochastic relocation, the introduction of such lending also makes the economy vulnerable to currency instability. We then show that these inflationary equilibria disappear when the lender of last resort either (i) imposes a borrowing constraint on banks that is sufficiently low or (ii) fixes the real interest rate on liquidity loans.

There are several directions in which the present analysis could be extended to address additional issues that figure prominently in discussions of the desirability and optimal design of lender-of-last-resort services. First, our model is set up so that the provision of loans to banks does not affect the government's intertemporal budget constraint. Yet the fiscal cost of bank bailouts is a primary concern in the design of lender-of-last-resort arrangements. Changes in the structure of the model could be made to address this issue. Second, it is often argued that the explicit or implicit access to loans provides banks with an incentive to take on "excessive" risk in its asset portfolio. This could be addressed by adding technologies to the model that give banks a choice regarding the riskiness of their investments. Third, in many emerging economies, and certainly in those that are contemplating dollarization, a large fraction of banks' liabilities and assets is denominated in foreign currency. Hence, the provision of lender-of-last-resort services, because of its effect on the money supply and thus on the exchange rate, may affect the real value of that part of the portfolio that is denominated in foreign exchange. Addressing this issue would require either a two-country or an open-economy version of the model. We leave all of these important issues for future research.

APPENDIX A: DERIVATION OF (8)

Using (7), the first-order condition can be written as

$$\pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi = \frac{p_{t+1}}{p_t} R_t \pi^* F(\pi^*) + \frac{\gamma_t}{1-\gamma_t} \int_{\pi^*}^{1} (1-\pi) f(\pi)\, d\pi,$$

or

$$\pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi = \frac{p_{t+1}}{p_t} R_t \pi^* F(\pi^*) + \frac{\gamma_t}{1-\gamma_t} \left\{ [1 - F(\pi^*)] - \int_{\pi^*}^{1} \pi f(\pi)\, d\pi \right\}.$$

If we add $\frac{\gamma_t}{1-\gamma_t}\pi^* F(\pi^*)$ to both sides, we have

$$\begin{aligned} &\pi^* F(\pi^*) + \frac{\gamma_t}{1-\gamma_t}\pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi = \\ &\left(\frac{p_{t+1}}{p_t} R_t + \frac{\gamma_t}{1-\gamma_t} \right) \pi^* F(\pi^*) + \frac{\gamma_t}{1-\gamma_t} \left\{ [1 - F(\pi^*)] - \int_{\pi^*}^{1} \pi f(\pi)\, d\pi \right\}, \end{aligned}$$

which reduces to

$$\pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi = \left[(1-\gamma_t) \frac{p_{t+1}}{p_t} R_t + \gamma_t \right] \pi^* F(\pi^*) + \gamma_t - \gamma_t F(\pi^*).$$

Making use of (7) again, we obtain

$$\pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi = \gamma_t F(\pi^*) + \gamma_t - \gamma_t F(\pi^*) = \gamma_t \tag{26}$$

This can be written in another form by noting that

$$\frac{d}{dx}[xF(x)] = F(x) + xf(x) \tag{27}$$

holds, which allows us to write

$$\begin{aligned} \int_{\pi^*}^{1} \pi f(\pi)\, d\pi &= \int_{\pi^*}^{1} \left\{ \frac{d}{d\pi}[\pi F(\pi)] - F(\pi) \right\} d\pi \\ &= \pi F(\pi)\Big|_{\pi^*}^{1} - \int_{\pi^*}^{1} F(\pi)\, d\pi \\ &= 1 - \pi^* F(\pi^*) - \int_{\pi^*}^{1} F(\pi)\, d\pi. \end{aligned}$$

This demonstrates that

$$1 - \int_{\pi^*}^{1} F(\pi)\, d\pi = \pi^* F(\pi^*) + \int_{\pi^*}^{1} \pi f(\pi)\, d\pi .$$

Substituting this into (26) yields the solution presented in (8).

APPENDIX B: PROOF OF PROPOSITION 1

We cannot solve (11) explicitly for γ_{t+1} as a function of γ_t, but we can derive some properties of the implicit function. First note that when $\gamma_{t+1} = 0$, which is its minimum value, γ_t is given by $\gamma_t = 1 - \int_0^1 F(\pi)\, d\pi = \int_0^1 \pi f(\pi)\, d\pi = E(\pi)$. Thus the implicit function is not defined for values of γ_t below the expected value of π. Next, when $\gamma_{t+1} = 1$, which is its maximum value, γ_t is strictly below one. Since the implicit function is continuous, there exists at least one steady state for γ. Moreover, at a steady state (11) implies that we have

$$\gamma = 1 - \int_{\frac{\gamma}{\gamma + \frac{y}{x}}}^{1} F(\pi)\, d\pi .$$

We know $F < 1$ always holds, so any steady state γ must satisfy

$$\gamma > 1 - \int_{\frac{\gamma}{\gamma + \frac{y}{x}}}^{1} d\pi = \frac{\gamma}{\gamma + \frac{y}{x}} .$$

Hence for any steady state

$$\gamma > 1 - \frac{y}{x} \tag{28}$$

must hold.

For the slope of (11), we can use Leibnitz's integral rule to obtain

$$\frac{d\gamma_t}{d\gamma_{t+1}} = F\left(\frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{y}{x}}\right) \frac{\frac{y}{x}}{\left(\gamma_{t+1} + \frac{y}{x}\right)^2} > 0 .$$

Thus γ_t as a function of γ_{t+1} is always increasing and hence is invertible. The inverse function is the law of motion for γ_t, and it is also strictly increasing. By the inverse function rule we obtain the slope of the law of motion,

$$\frac{d\gamma_{t+1}}{d\gamma_t} = \frac{\left(\gamma_{t+1} + \frac{y}{\bar{x}}\right)^2}{\frac{y}{\bar{x}}} \frac{1}{F\left(\frac{\gamma_{t+1}}{\gamma_{t+1}+\frac{y}{\bar{x}}}\right)} > 0.$$

Suppose we evaluate this slope at any steady state. Given (28) and taking into account that the economy is a Samuelson case economy, which implies that $y < \bar{x}$, the first term of the slope is greater than one for any steady state. The second term is always greater than or equal to one, so for any steady state the derivative itself must be greater than one. The law of motion for γ_t must therefore cross the 45° line from below at every steady state. This implies that there is exactly one steady state, which we will denote by γ_a and that this steady state is in the open interval $(E(\pi); \infty)$. The steady state is unstable, and all nonstationary trajectories eventually leave the feasible region. Hence the steady state is the only equilibrium of this economy. ■

APPENDIX C: DERIVATION OF (19)

Using the definition of π^* given by (7), the first order condition can be rewritten as:

$$(1 - R_t \frac{p_{t+1}}{p_t}) \frac{\pi^*}{\gamma_t} F(\pi^{**}) + \frac{1}{\gamma_t + c} \int_{\pi^{**}}^{\infty} \pi f(\pi) d\pi = \frac{R_t}{(1-\gamma_t) R_t - c\frac{p_t}{p_{t+1}}} \int_{\pi^{**}}^{\infty} (1-\pi) f(\pi) d\pi. \quad (29)$$

Now, note that (7) and (18) imply that we have

$$\frac{1}{\gamma_t + c} = \frac{\pi^*}{\pi^{**}} \frac{1}{\gamma_t}$$

and

$$\frac{R_t}{(1-\gamma_t) R_t - c\frac{p_t}{p_{t+1}}} = \frac{\pi^*}{1-\pi^{**}} R_t \frac{p_{t+1}}{p_t} \frac{1}{\gamma_t}.$$

This allows us to write (29) as

$$(1 - R_t \frac{p_{t+1}}{p_t}) \pi^{**} F(\pi^{**}) + \int_{\pi^{**}}^{\infty} \pi f(\pi) d\pi = \frac{\pi^{**}}{1-\pi^{**}} R_t \frac{p_{t+1}}{p_t} \int_{\pi^{**}}^{\infty} (1-\pi) f(\pi) d\pi. \quad (30)$$

Using (27), note that we have

$$\begin{aligned}
\int_{\pi^{**}}^{1} \pi f(\pi)\, d\pi &= \int_{\pi^{**}}^{1} \left\{ \frac{d}{d\pi}\left[\pi F(\pi)\right] - F(\pi) \right\} d\pi \\
&= \pi F(\pi)\,\Big|_{\pi^{**}}^{1} - \int_{\pi^{**}}^{1} F(\pi)\, d\pi \\
&= 1 - \pi^{**} F(\pi^{**}) - \int_{\pi^{**}}^{1} F(\pi) d\pi.
\end{aligned}$$

Hence, (30) reduces to

$$1 - \int_{\pi^{**}}^{1} F(\pi) d\pi = \frac{\pi^{**}}{1 - \pi^{**}} R_t \frac{p_{t+1}}{p_t} \int_{\pi^{**}}^{1} F(\pi) d\pi. \tag{31}$$

Multiplying (31) by $(1 - \pi^{**})$ and rearranging gives

$$1 - \pi^{**} = \left(1 - \pi^{**} + \pi^{**} R_t \frac{p_{t+1}}{p_t}\right) \int_{\pi^{**}}^{1} F(\pi) d\pi. \tag{32}$$

But (18) implies that we have

$$\pi^{**} R_t \frac{p_{t+1}}{p_t} = \frac{\gamma_t (1 - \pi^{**}) + c}{(1 - \gamma_t)}.$$

Therefore, (32) becomes

$$1 - \pi^{**} = \left[1 - \pi^{**} + \frac{\gamma_t (1 - \pi^{**}) + c}{(1 - \gamma_t)}\right] \int_{\pi^{**}}^{1} F(\pi) d\pi. \tag{33}$$

Multiplying (33) $(1 - \gamma_t)$ and rearranging terms yields

$$1 - \gamma_t = \left[\frac{(1 - \gamma_t)(1 - \pi^{**}) + \gamma_t (1 - \pi^{**}) + c}{(1 - \pi^{**})}\right] \int_{\pi^{**}}^{1} F(\pi) d\pi.$$

This clearly reduces to (19).

APPENDIX D: DERIVATION OF (24)

Using (7) and (23) in the first-order condition, we obtain

$$\mu^*F(\mu^*) + \mu^{**}[1-F(\mu^{**})] + \int_{\mu^*}^{\mu^{**}} \mu f(\mu)\,d\mu =$$
$$\frac{p_{t+1}}{p_t}R_t\mu^*F(\mu^*) + R_t\mu^{**}[1-F(\mu^{**})] + \frac{\gamma_t}{1-\gamma_t}\int_{\mu^*}^{\mu^{**}} (1-\mu)f(\mu)\,d\mu;$$

or

$$\mu^*F(\mu^*) + \mu^{**}[1-F(\mu^{**})] + \int_{\mu^*}^{\mu^{**}} \mu f(\mu)\,d\mu =$$
$$\frac{p_{t+1}}{p_t}R_t\mu^*F(\mu^*) + R_t\mu^{**}[1-F(\mu^{**})] + \frac{\gamma_t}{1-\gamma_t}\left\{[F(\mu^{**})-F(\mu^*)] - \int_{\mu^*}^{\mu^{**}} \mu f(\mu)\,d\mu\right\}.$$

If we add $\frac{\gamma_t}{1-\gamma_t}\mu^*F(\mu^*) + \frac{\gamma_t}{1-\gamma_t}\mu^{**}[1-F(\mu^{**})]$ to both sides, we have

$$\mu^*F(\mu^*) + \mu^{**}[1-F(\mu^{**})] + \int_{\mu^*}^{\mu^{**}} \mu f(\mu)\,d\mu + \frac{\gamma_t}{1-\gamma_t}\mu^*F(\mu^*) + \frac{\gamma_t}{1-\gamma_t}\mu^{**}[1-F(\mu^{**})] =$$
$$\left(\frac{p_{t+1}}{p_t}R_t + \frac{\gamma_t}{1-\gamma_t}\right)\mu^*F(\mu^*) + \left(R_t + \frac{\gamma_t}{1-\gamma_t}\right)\mu^{**}[1-F(\mu^{**})] +$$
$$\frac{\gamma_t}{1-\gamma_t}\left\{[F(\mu^{**})-F(\mu^*)] - \int_{\mu^*}^{\mu^{**}} \mu f(\mu)\,d\mu\right\}.$$

This reduces to

$$\mu^*F(\mu^*) + \mu^{**}[1-F(\mu^{**})] + \int_{\mu^*}^{\mu^{**}} \mu f(\mu)\,d\mu =$$
$$\left[(1-\gamma_t)\frac{p_{t+1}}{p_t}R_t + \gamma_t\right]\mu^*F(\mu^*) + [(1-\gamma_t)R_t + \gamma_t]\mu^{**}[1-F(\mu^{**})] + \gamma_t[F(\mu^{**})-F(\mu^*)].$$

Making use of (7) and (23) again, we obtain

$$\mu^*F(\mu^*) + \mu^{**}[1-F(\mu^{**})] + \int_{\mu^*}^{\mu^{**}} \mu f(\mu)\,d\mu$$
$$= \gamma_t F(\mu^*) + \gamma_t[1-F(\mu^{**})] + \gamma_t F(\mu^{**}) - \gamma_t F(\mu^*)$$

Therefore, the solution to the problem is

$$\gamma_t = \mu^*F(\mu^*) + \mu^{**}[1-F(\mu^{**})] + \int_{\mu^*}^{\mu^{**}} \mu f(\mu)\,d\mu. \qquad (34)$$

Using (27), note that we have

$$\begin{aligned}
\int_{\pi^*}^{\pi^{**}} \pi f(\pi)\, d\pi &= \int_{\pi^*}^{\pi^{**}} \left\{ \frac{d}{d\pi}[\pi F(\pi)] - F(\pi) \right\} d\pi \\
&= \pi F(\pi)\big|_{\pi^*}^{\pi^{**}} - \int_{\pi^*}^{\pi^{**}} F(\pi)\, d\pi \\
&= \pi^{**} F(\pi^{**}) - \pi^* F(\pi^*) - \int_{\pi^*}^{\pi^{**}} F(\pi)\, d\pi.
\end{aligned}$$

Therefore, it is the case that

$$-\int_{\pi^*}^{\pi^{**}} F(\pi)\, d\pi = \pi^* F(\pi^*) - \pi^{**} F(\pi^{**}) + \int_{\pi^*}^{\pi^{**}} \pi f(\pi)\, d\pi$$

holds. Substituting this into (34) yields the solution presented in (24).

APPENDIX E: PROOF OF PROPOSITION 4

We divide the phase plane into two regions and analyze the equilibrium law of motion in each region. When $\gamma_{t+1} \geq \gamma_t$, we know that the law of motion is the same as in Section 4, and therefore no equilibrium has $\gamma_{t+1} > \gamma_t$ for some t. The only place the law of motion lies on the $\gamma_{t+1} = \gamma_t$ ray (and hence the only steady-state equilibrium) is $\gamma_t = 1 - \frac{y}{x} = \gamma_b$ for all t. This is exactly the same allocation as the steady state in section 4, and is therefore Pareto optimal.

What remains is to show that there are no equilibria where $\gamma_{t+1} < \gamma_t$ for some t. The law of motion in this region is implicitly defined by (25). The slope of this curve is given by

$$\frac{d\gamma_{t+1}}{d\gamma_t} = \frac{1 - \frac{\frac{y}{x}}{(\gamma_t + \frac{y}{x})^2}\left[1 - F\left(\frac{\gamma_t}{\gamma_t + \frac{y}{x}}\right)\right]}{\frac{\frac{y}{x}}{(\gamma_{t+1} + \frac{y}{x})^2} F\left(\frac{\gamma_{t+1}}{\gamma_{t+1} + \frac{y}{x}}\right)}.$$

The implicit function is defined and continuous in a neighborhood of any point where the denominator of this expression is nonzero, that is, where γ_{t+1} is nonzero. Hence the law of motion is only defined over values of γ_t such that the implied γ_{t+1} is positive. Such values of γ_t must satisfy

$$\begin{aligned}
\gamma_t &> \frac{\gamma_t}{\gamma_t + \frac{y}{x}} - \int_0^{\frac{\gamma_t}{\gamma_t + \frac{y}{x}}} F(\pi)\, d\pi \\
&= \frac{\gamma_t}{\gamma_t + \frac{y}{x}}\left(1 - F\left(\frac{\gamma_t}{\gamma_t + \frac{y}{x}}\right)\right) + \int_0^{\frac{\gamma_t}{\gamma_t + \frac{y}{x}}} \pi f(\pi)\, d\pi.
\end{aligned}$$

Hence such values of γ_t must satisfy

$$\gamma_t > \frac{\gamma_t}{\gamma_t + \frac{y}{\bar{x}}}\left(1 - F\left(\frac{\gamma_t}{\gamma_t + \frac{y}{\bar{x}}}\right)\right)$$

so that we have

$$\frac{\frac{y}{\bar{x}}}{\left(\gamma_t + \frac{y}{\bar{x}}\right)^2}\left(1 - F\left(\frac{\gamma_t}{\gamma_t + \frac{y}{\bar{x}}}\right)\right) < \frac{\gamma_t}{\gamma_t + \frac{y}{\bar{x}}}\left(\frac{\frac{y}{\bar{x}}}{\gamma_t}\right) = \frac{\frac{y}{\bar{x}}}{\gamma_t + \frac{y}{\bar{x}}} < 1;$$

and the numerator of the slope is positive. This demonstrates that wherever the law of motion is defined, it is strictly increasing.

Note that from (25) we have

$$\lim_{\gamma_t \to 1 - \frac{y}{\bar{x}}} \gamma_{t+1} = 1 - \frac{y}{\bar{x}}.$$

Therefore the law of motion is (left) continuous at $\gamma_t = 1 - \frac{y}{\bar{x}}$. The slope of the law of motion is greater than unity at this point, and therefore is not defined for $\gamma_t > 1 - \frac{y}{\bar{x}}$. It cannot intersect the 45-degree line anywhere else, because continuity would then imply that there are multiple steady-state equilibria, contradicting the results above. Therefore the law of motion is a strictly increasing function defined on an interval $\left(\bar{\gamma}, 1 - \frac{y}{\bar{x}}\right]$; where $\bar{\gamma}$ is the largest solution to the equation

$$\gamma = \frac{\gamma}{\gamma + \frac{y}{\bar{x}}} - \int_0^{\frac{\gamma}{\gamma + \frac{y}{\bar{x}}}} F(\pi)\, d\pi \equiv g(\gamma) \tag{35}$$

(this is the value of γ_t that would imply $\gamma_{t+1} = 0$). We now proceed to show that $\bar{\gamma} > 0$ holds. The function g is defined for all $\gamma \geq 0$ and is continuous.. The first two derivatives are

$$\begin{aligned} g'(\gamma) &= \frac{\frac{y}{\bar{x}}}{\left(\gamma_t + \frac{y}{\bar{x}}\right)^2}\left(1 - F\left(\frac{\gamma_t}{\gamma_t + \frac{y}{\bar{x}}}\right)\right) > 0 \\ g''(\gamma) &= -2\frac{\frac{y}{\bar{x}}}{\left(\gamma_t + \frac{y}{\bar{x}}\right)^3}\left(1 - F\left(\frac{\gamma_t}{\gamma_t + \frac{y}{\bar{x}}}\right)\right) - \frac{\frac{y}{\bar{x}}}{\left(\gamma_t + \frac{y}{\bar{x}}\right)^2} f\left(\frac{\gamma_t}{\gamma_t + \frac{y}{\bar{x}}}\right)\frac{\frac{y}{\bar{x}}}{\gamma_t + \frac{y}{\bar{x}}} < 0. \end{aligned}$$

This shows that the function g is strictly increasing and strictly concave. It begins at the origin, with slope equal to $\frac{\bar{x}}{y} > 1$ at this point. There is therefore a unique positive solution to (35); let $\bar{\gamma}$

denote this solution. Note that we have

$$g\left(1 - \frac{y}{x}\right) = 1 - \frac{y}{x} - \int_0^{1-\frac{y}{x}} F(\pi)\, d\pi < 1 - \frac{y}{x};$$

so that $\theta < 1 - \frac{y}{x}$ holds.

The above analysis demonstrates that the law of motion is an increasing function on $\left(\theta, 1 - \frac{y}{x}\right]$ for some $\theta > 0$; and is not defined for $\gamma_t < \theta$. This curve is depicted in Fig. 4. The analysis implies that any trajectory with $\gamma_{t+1} < \gamma_t$ for some t will leave the feasible region in finite time, and therefore cannot be an equilibrium. Therefore $\gamma_t = \gamma_b$ for all t is the unique equilibrium. ■

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